Chapter 12

Concentrations of Solutions

SOLUTE AND SOLVENT

In a solution of one substance in another substance, the dissolved substance is called the *solute*. The substance in which the solute is dissolved is called the *solvent*. When the relative amount of one substance in a solution is much greater than that of the other, the substance present in greater amount is generally regarded as the solvent. When the relative amounts of the two substances are of the same order of magnitude, it becomes difficult, in fact arbitrary, to specify which substance is the solvent.

CONCENTRATIONS EXPRESSED IN PHYSICAL UNITS

When physical units are employed, the concentrations of solutions are generally expressed in one of the following ways.

(1) By the mass of solute per unit volume of solution (e.g. 20 grams of KCl per liter of solution).

(2) By the percentage composition, or the number of mass units of solute per 100 mass units of solution.

EXAMPLE 1 A 10% aqueous NaCl solution contains 10 g of NaCl per 100 g of solution. Ten grams of NaCl is mixed with 90 g of water to form 100 g of solution.

(3) By the mass of solute per mass of solvent (e.g. 5.2 g of NaCl in 100 g of water).

CONCENTRATIONS EXPRESSED IN CHEMICAL UNITS

Molar Concentration

The *molar concentration* (M) is the number of moles of the solute contained in one liter of solution.

EXAMPLE 2 A 0.5 molar (0.5 M) solution of H_2SO_4 contains 49.04 g of H_2SO_4 per liter of solution, since 49.04 is half of the molecular weight of H_2SO_4, 98.08. A one molar (1 M) solution contains 98.08 g H_2SO_4 per liter of solution.

Note that M is the symbol for a quantity, the molar concentration, and M is the symbol for a unit, mol/L. (The term *molarity* is often used to refer to molar concentration; it will not be used in this book, in order to avoid confusion with *molality*, defined below.)

Normality

The *normality* of a solution (N) is the number of *gram-equivalents* of the solute contained in one liter of solution. The *equivalent weight* is that fraction of the molecular weight which corresponds to one defined unit of chemical reaction, and a *gram-equivalent* is this same fraction of a mole. (The gram-equivalent is referred to in some books as the *equivalent* or the *gram-equivalent weight*.) Equivalent weights are determined as follows.

(1) The defined unit of reaction for acids and bases is the neutralization reaction

$$H^+ + OH^- \rightarrow H_2O$$

The equivalent weight of an acid is that fraction of the molecular weight which contains or can supply for reaction one acid H^+. (In other words, the equivalent weight is the molecular weight divided by the number of H^+ furnished per molecule.) A gram-equivalent (g-eq) is then that amount which contains or can supply for reaction 1 mol of H^+.

EXAMPLE 3 The equivalent weights of HCl and $HC_2H_3O_2$ are the same as their molecular weights, since each contains one acidic hydrogen per molecule. A g-eq of each of these molecules is the same as 1 mol. The equivalent weight of H_2SO_4 is usually half the molecular weight and a g-eq is $\frac{1}{2}$ mol, since both hydrogens are replaceable in most reactions of dilute sulfuric acid. A g-eq of H_3PO_4 may be 1 mol, $\frac{1}{2}$ mol, or $\frac{1}{3}$ mol, depending on whether 1, 2, or 3 hydrogen atoms per molecule are replaced in a particular reaction. A g-eq of H_3BO_3 is always 1 mol, since only one hydrogen is replaceable in neutralization reactions. The equivalent weight of SO_3 is one-half the molecular weight, since SO_3 can react with water to give two H^+.

$$SO_3 + H_2O \rightarrow 2H^+ + SO_4^{2-}$$

There are no simple rules for predicting how many hydrogens of an acid will be replaced in a given neutralization.

(2) The equivalent weight of a base is that fraction of the formula weight which contains or can supply one OH^-, or can react with one H^+.

EXAMPLE 4 The equivalent weights of NaOH, NH_3 (which can react with water to give $NH_4^+ + OH^-$), $Mg(OH)_2$, and $Al(OH)_3$, are equal to 1/1, 1/1, 1/2, and 1/3 of their formula weights, respectively.

(3) The equivalent weight of an oxidizing or reducing agent for a particular reaction is equal to its formula weight divided by the total number of electrons gained or lost when the reaction of that formula unit occurs. A given oxidizing or reducing agent may have more than one equivalent weight, depending on the reaction for which it is used.

Equivalent weights are so defined that equal numbers of gram-equivalents of two substances react exactly with each other. This is true for neutralization because one H^+ neutralizes one OH^-, and for oxidation-reduction because the number of electrons lost by the reducing agent equals the number of electrons gained by the oxidizing agent.

EXAMPLE 5 1 mol HCl, $\frac{1}{2}$ mol H_2SO_4, and $\frac{1}{6}$ mol $K_2Cr_2O_7$ (as oxidizing agent), each in 1 L of solution, give normal (1 N) solutions of these substances. A normal (1 N) solution of HCl is also a molar (1 M) solution. A normal (1 N) solution of H_2SO_4 is also a one-half molar (0.5 M) solution.

Note that N is the symbol for a quantity, the normality, and N is the symbol for a unit, g-eq/L.

Molality

The *molality* of a solution is the number of moles of the solute per kilogram of solvent contained in a solution. The molality (m) cannot be computed from the molar concentration (M) unless the density of the solution is known (see Problem 12.58).

EXAMPLE 6 A solution made up of 98.08 g of pure H_2SO_4 and 1000 g of water would be a 1 molal solution of H_2SO_4. (Some books use the symbol m to designate both the quantity, the molality, and the unit, mol/kg. In this book, m is used to refer to the quantity only and not to the unit.)

Mole Fraction

The *mole fraction* (x) of any component in a solution is defined as the number of moles (n) of that component, divided by the total number of moles of all components in the solution. The sum of the mole fractions of all components of a solution is 1. In a two-component solution,

$$x(\text{solute}) = \frac{n(\text{solute})}{n(\text{solute}) + n(\text{solvent})} \qquad x(\text{solvent}) = \frac{n(\text{solvent})}{n(\text{solute}) + n(\text{solvent})}$$

Expressed as a percentage, mole fraction is called *mole percent*.

COMPARISON OF THE CONCENTRATION SCALES

The *molar concentration* and *normality* scales are useful for volumetric experiments, in which the amount of solute in a given portion of solution is related to the measured volume of solution. As will be seen in subsequent chapters, the normality scale is very convenient for comparing the relative volumes required for two solutions to react chemically with each other. A limitation of the normality scale is that a given solution may have more than one normality, depending on the reaction for which it is used. The molar concentration of a solution, on the other hand, is a fixed number because the molecular weight of a substance does not depend on the reaction for which the substance is used, as may the equivalent weight.

The *molality* scale is useful for experiments in which physical measurements (as of freezing point, boiling point, vapor pressure, etc.) are made over a wide range of temperatures. The molality of a given solution, which is determined solely by the masses of the solution components, is independent of temperature. The molar concentration or the normality of a solution, on the other hand, being defined in terms of the volume, may vary appreciably as the temperature is changed, because of the temperature dependence of the volume.

The *mole fraction* scale finds use in theoretical work because many physical properties of solutions (Chapter 14) are expressed most clearly in terms of the relative numbers of solvent and solute molecules. (The number of moles of a substance is proportional to the number of molecules.)

SUMMARY OF CONCENTRATION UNITS

$$\text{molar concentration of a solution} = M = \frac{\text{number of moles of solute}}{\text{number of liters of solution}}$$

$$\text{normality of a solution} = N = \frac{\text{number of gram-equivalents of solute}}{\text{number of liters of solution}}$$

$$= \frac{\text{number of milliequivalents of solute}}{\text{number of cubic centimeters of solution}}$$

$$\text{molality of a solution} = m = \frac{\text{number of moles of solute}}{\text{number of kilograms of solvent}}$$

$$\text{mole fraction of any component} = x = \frac{\text{number of moles of that component}}{\text{total number of moles of all components}}$$

The second expression for the normality is obtained from the first expression by multiplying numerator and denominator by 1000.

DILUTION PROBLEMS

The volumetric scales of concentration are those, like molar concentration and normality, in which the concentration is expressed as the amount of solute per fixed volume of solution. When the concentration is expressed on a volumetric scale, the amount of solute contained in a given volume of solution is equal to the product of the volume and the concentration:

$$\text{amount of solute} = \text{volume} \times \text{concentration}$$

If a solution is diluted, the volume is increased and the concentration is decreased, but the total amount of solute is constant. Hence, two solutions of different concentrations but containing the same amounts of solute will be related to each other as follows:

$$\text{volume}_1 \times \text{concentration}_1 = \text{volume}_2 \times \text{concentration}_2$$

If any three terms in the above equation are known, the fourth can be calculated. The quantities on both sides of the equation must be expressed in the same units.

Solved Problems

CONCENTRATIONS EXPRESSED IN PHYSICAL UNITS

12.1. Explain how you would prepare 60 cm^3 of an aqueous solution of $AgNO_3$ of strength 0.030 g $AgNO_3$ per cm^3.

Since each cm^3 of solution is to contain 0.030 g $AgNO_3$, 60 cm^3 of solution should contain

$$(0.030 \text{ g/cm}^3)(60 \text{ cm}^3) = 1.8 \text{ g } AgNO_3$$

Thus, dissolve 1.8 g $AgNO_3$ in about 50 cm^3 of water. Then add sufficient water to make the volume exactly 60 cm^3. Stir thoroughly to insure uniformity. (Note that 60 cm^3 is to be the volume of the final solution, not of the water used to make up the solution.)

12.2. How many grams of a 5.0% by weight NaCl solution are necessary to yield 3.2 g NaCl?

A 5.0% NaCl solution contains 5.0 g NaCl in 100 g of solution. Then:

$$1 \text{ g NaCl} \quad \text{is contained in} \quad \frac{100}{5.0} \text{ g solution}$$

and

$$3.2 \text{ g NaCl} \quad \text{is contained in} \quad (3.2)\left(\frac{100}{5.0} \text{ g solution}\right) = 64 \text{ g solution}$$

Or, by proportion, letting x = required number of grams of solution,

$$\frac{5.0 \text{ g NaCl}}{100 \text{ g solution}} = \frac{3.2 \text{ g NaCl}}{x} \qquad \text{whence} \qquad x = 64 \text{ g solution}$$

Or, by the use of a quantitative factor,

$$x \text{ g solution} = (3.2 \text{ g NaCl})\left(\frac{100 \text{ g solution}}{5.0 \text{ g NaCl}}\right) = 64 \text{ g solution}$$

12.3. How much $NaNO_3$ must be weighed out to make 50 cm^3 of an aqueous solution containing 70 mg Na^+ per cm^3?

$$\text{mass of } Na^+ \text{ in 50 cm}^3 \text{ of solution} = (50 \text{ cm}^3)(70 \text{ mg/cm}^3) = 3500 \text{ mg} = 3.5 \text{ g } Na^+$$

Formula weight of $NaNO_3$ is 85; atomic weight of Na is 23. Then:

$$23 \text{ g } Na^+ \quad \text{is contained in} \quad 85 \text{ g } NaNO_3$$

$$1 \text{ g } Na^+ \quad \text{is contained in} \quad \frac{85}{23} \text{ g } NaNO_3$$

and

$$3.5 \text{ g } Na^+ \quad \text{is contained in} \quad (3.5)\left(\frac{85}{23} \text{ g}\right) = 12.9 \text{ g } NaNO_3$$

Or, by direct use of quantitative factors,

$$x \text{ g } NaNO_3 = (50 \text{ cm}^3 \text{ solution})\left(\frac{70 \text{ mg } Na^+}{1 \text{ cm}^3 \text{ solution}}\right)\left(\frac{85 \text{ g } NaNO_3}{23 \text{ g } Na^+}\right)\left(\frac{1 \text{ g}}{1000 \text{ mg}}\right) = 12.9 \text{ g } NaNO_3$$

12.4. Calculate the mass of $Al_2(SO_4)_3 \cdot 18H_2O$ needed to make up 50 cm^3 of an aqueous solution of strength 40 mg Al^{3+} per cm^3.

$$\text{mass of } Al^{3+} \text{ in 50 cm}^3 \text{ of solution} = (50 \text{ cm}^3)(40 \text{ mg/cm}^3) = 2000 \text{ mg} = 2.00 \text{ g } Al^{3+}$$

Atomic weight of Al is 27; formula weight of $Al_2(SO_4)_3 \cdot 18H_2O$ is 666.

$$x \text{ g } Al_2(SO_4)_3 \cdot 18H_2O = (2.00 \text{ g } Al^{3+})\left(\frac{666 \text{ g } Al_2(SO_4)_3 \cdot 18H_2O}{54 \text{ g } Al^{3+}}\right) = 24.7 \text{ g } Al_2(SO_4)_3 \cdot 18H_2O$$

A solution of identical composition could be prepared from the appropriate amount of anhydrous $Al_2(SO_4)_3$. To compute the appropriate amount in this case, the formula weight of $Al_2(SO_4)_3$, 342, would be used instead of 666 above. The experimenter would find that more water is needed to make up the 50 cm^3 of solution from the anhydrous salt than from the hydrate. In general, hydrated salts differ from the anhydrous salts only in the crystalline state. In solution, the water of hydration and the water of the solvent become indistinguishable from each other.

12.5. Describe how you would prepare 50 g of a 12.0% $BaCl_2$ solution, starting with $BaCl_2 \cdot 2H_2O$ and pure water.

A 12.0% $BaCl_2$ solution contains 12.0 g $BaCl_2$ per 100 g of solution, or 6.0 g $BaCl_2$ in 50 g of solution. Formula weight of $BaCl_2$ is 208; of $BaCl_2 \cdot 2H_2O$, 244. Therefore,

$$208 \text{ g } BaCl_2 \quad \text{is contained in} \quad 244 \text{ g } BaCl_2 \cdot 2H_2O$$

$$1 \text{ g } BaCl_2 \quad \text{is contained in} \quad \frac{244}{208} \text{ g } BaCl_2 \cdot 2H_2O$$

and

$$6.0 \text{ g } BaCl_2 \quad \text{is contained in} \quad (6.0)\left(\frac{244}{208}\text{ g}\right) = 7.0 \text{ g } BaCl_2 \cdot 2H_2O$$

The desired solution is prepared by dissolving 7.0 g $BaCl_2 \cdot 2H_2O$ in 43 g (43 cm^3) of water.

12.6. Calculate the mass of anhydrous HCl in 5.00 cm^3 of concentrated hydrochloric acid (density 1.19 g/cm^3) containing 37.23% HCl by weight.

The mass of 5.00 cm^3 of solution is $(5.00\text{ cm}^3)(1.19\text{ g/cm}^3) = 5.95\text{ g}$. The solution contains 37.23% HCl by weight. Hence the mass of HCl in 5.95 g solution is

$$(0.3723)(5.95\text{ g}) = 2.22\text{ g anhydrous HCl}$$

12.7. Calculate the volume of concentrated sulfuric acid (density 1.84 g/cm^3), containing 98% H_2SO_4 by weight, that would contain 40.0 g of pure H_2SO_4.

One cm^3 of solution has a mass of 1.84 g and contains $(0.98)(1.84\text{ g}) = 1.80\text{ g}$ of pure H_2SO_4. Then 40.0 g H_2SO_4 is contained in

$$\left(\frac{40.0}{1.80}\right)(1\text{ cm}^3\text{ solution}) = 22.2\text{ cm}^3\text{ solution}$$

12.8. Exactly 4.00 g of a solution of sulfuric acid was diluted with water, and an excess of $BaCl_2$ was added. The washed and dried precipitate of $BaSO_4$ weighed 4.08 g. Find the percent H_2SO_4 in the original acid solution.

First determine the mass of H_2SO_4 required to precipitate 4.08 g $BaSO_4$ by the reaction

$$H_2SO_4 + BaCl_2 \rightarrow 2HCl + BaSO_4$$

The equation shows that 1 mol $BaSO_4$ (233.4 g) requires 1 mol H_2SO_4 (98.08 g). Therefore, 4.08 g $BaSO_4$ requires

$$\left(\frac{4.08\text{ g } BaSO_4}{233.4\text{ g } BaSO_4}\right)(98.08\text{ g } H_2SO_4) = 1.72\text{ g } H_2SO_4$$

and

$$\text{fraction } H_2SO_4 \text{ by weight} = \frac{\text{mass of } H_2SO_4}{\text{mass of solution}} = \frac{1.72\text{ g}}{4.00\text{ g}} = 0.430 = 43.0\%$$

CONCENTRATIONS EXPRESSED IN CHEMICAL UNITS

12.9. How many grams of solute are required to prepare 1 liter of 1 M $Pb(NO_3)_2$? What is the molar concentration of the solution with respect to each of the ions?

A molar solution contains 1 mol of solute in 1 L of solution. The formula weight of $Pb(NO_3)_2$ is 331.2; hence 331.2 g of $Pb(NO_3)_2$ is needed.

A 1 M solution of $Pb(NO_3)_2$ is 1 M with respect to Pb^{2+} and 2 M with respect to NO_3^-.

12.10. What is the molar concentration of a solution containing 16.0 g CH_3OH in 200 cm^3 of solution?

Molecular weight of CH_3OH is 32.0.

$$M = \text{molar concentration} = \frac{n(\text{solute})}{\text{volume of solution in L}} = \frac{\dfrac{16.0\text{ g}}{32.0\text{ g/mol}}}{0.200\text{ L}} = 2.50\text{ mol/L} = 2.50\text{ M}$$

12.11. Determine the molar concentration of each of the following solutions: (*a*) 18.0 g $AgNO_3$ per liter of solution, (*b*) 12.00 g $AlCl_3 \cdot 6H_2O$ per liter of solution. Formula weight of $AgNO_3$ is 169.9; of $AlCl_3 \cdot 6H_2O$, 241.4.

(*a*) $$\frac{18.0\text{ g/L}}{169.9\text{ g/mol}} = 0.106\text{ mol/L} = 0.106\text{ M}$$

(*b*) $$\frac{12.00\text{ g/L}}{241.4\text{ g/mol}} = 0.0497\text{ mol/L} = 0.0497\text{ M}$$

12.12. How much $(NH_4)_2SO_4$ is required to prepare 400 cm^3 of M/4 solution? (The notation M/4 is sometimes used in place of $\frac{1}{4}$ M.)

Formula weight of $(NH_4)_2SO_4$ is 132.1. One liter of M/4 solution contains $\frac{1}{4}(132.1\text{ g}) = 33.02\text{ g}$ $(NH_4)_2SO_4$. Then 400 cm^3 of M/4 solution requires

$$(0.400\text{ L})(33.02\text{ g/L}) = 13.21\text{ g } (NH_4)_2SO_4$$

Another Method

$$\begin{aligned} \text{mass} &= (\text{molar concentration}) \times (\text{formula weight}) \times (\text{volume of solution}) \\ &= (\tfrac{1}{4}\text{ mol/L})(132.1\text{ g/mol})(0.400\text{ L}) = 13.21\text{ g } (NH_4)_2SO_4 \end{aligned}$$

12.13. What is the molality of a solution which contains 20.0 g of cane sugar, $C_{12}H_{22}O_{11}$, dissolved in 125 g of water?

Molecular weight of $C_{12}H_{22}O_{11}$ is 342.

$$m = \text{molality} = \frac{n(\text{solute})}{\text{mass of solvent in kg}} = \frac{(20.0\text{ g})/(342\text{ g/mol})}{0.125\text{ kg}} = 0.468\text{ mol/kg}$$

12.14. The molality of a solution of ethyl alcohol, C_2H_5OH, in water is 1.54 mol/kg. How many grams of alcohol are dissolved in 2.5 kg of water?

Molecular weight of C_2H_5OH is 46.1. Since the molality is 1.54, 1 kg water dissolves 1.54 mol alcohol. Then 2.5 kg water dissolves $(2.5)(1.54) = 3.85$ mol alcohol, and

$$\text{mass of alcohol} = (3.85\text{ mol})(46.1\text{ g/mol}) = 177\text{ g alcohol}$$

12.15. Calculate the (*a*) molar concentration and (*b*) molality of a sulfuric acid solution of density 1.198 g/cm^3, containing 27.0% H_2SO_4 by weight.

(*a*) Each cm^3 of acid solution has a mass of 1.198 g and contains (0.270) (1.198) = 0.324 g H_2SO_4. Since the molecular weight of H_2SO_4 is 98.1,

$$M = \frac{n(H_2SO_4)}{\text{volume of solution in L}} = \frac{(0.324\ g)/(98.1\ g/mol)}{(1\ cm^3)(10^{-3}\ L/cm^3)} = 3.30\ mol/L = 3.30\ M$$

(*b*) From (*a*), there is 324 g, or 3.30 mol, of solute per liter of solution. The amount of water in one liter of solution is 1198 g − 324 g = 874 g H_2O. Hence

$$m = \frac{n(\text{solute})}{\text{mass of solvent in kg}} = \frac{3.30\ mol\ H_2SO_4}{0.874\ kg\ H_2O} = 3.78\ mol/kg$$

12.16. Determine the mole fractions of both substances in a solution containing 36.0 g of water and 46 g of glycerin, $C_3H_5(OH)_3$.

Molecular weight of $C_3H_5(OH)_3$ is 92; of H_2O, 18.0.

$$n(\text{glycerin}) = \frac{46\ g}{92\ g/mol} = 0.50\ mol \qquad n(\text{water}) = \frac{36.0\ g}{18.0\ g/mol} = 2.00\ mol$$

$$\text{total number of moles} = 0.50 + 2.00 = 2.50\ mol$$

$$x(\text{glycerin}) = \text{mole fraction of glycerin} = \frac{n(\text{glycerin})}{\text{total number of moles}} = \frac{0.50}{2.50} = 0.20$$

$$x(\text{water}) = \text{mole fraction of water} = \frac{n(\text{water})}{\text{total number of moles}} = \frac{2.00}{2.50} = 0.80$$

Check: sum of mole fractions = 0.20 + 0.80 = 1.00.

12.17. How many gram-equivalents of solute are contained in (*a*) 1 liter of 2 N solution, (*b*) 1 liter of 0.5 N solution, (*c*) 0.5 liter of 0.2 N solution?

A normal solution contains 1 g-eq of solute in 1 L of solution.

(*a*) 1 L of 2 N contains 2 g-eq of solute.

(*b*) 1 L of 0.5 N contains 0.5 g-eq of solute.

(*c*) 0.5 L of 0.2 N contains (0.5 L) (0.2 g-eq/L) = 0.1 g-eq of solute.

12.18. How many (*a*) gram-equivalents and (*b*) milliequivalents of solute are present in 60 cm^3 of 4.0 N solution?

(*a*) number of gram-equivalents = (number of liters) × (normality) = (0.060 L) (4.0 g-eq/L) = 0.24 g-eq

(*b*) (0.24 g-eq) (1000 meq/g-eq) = 240 meq

Another Method

number of meq = (number of cm^3) × (normality) = (60 cm^3) (4.0 meq/cm^3) = 240 meq

12.19. How many grams of solute are required to prepare 1 L of 1 N solution of (*a*) LiOH, (*b*) Br_2 (as oxidizing agent), (*c*) H_3PO_4 (for a reaction in which three H are replaceable)?

A normal solution contains 1 g-eq of solute in 1 L of solution. Formula weight of LiOH is 23.95; of Br_2, 159.82; of H_3PO_4, 97.99.

(*a*) One liter of 1 N LiOH requires (23.95/1) g = 23.95 g LiOH.

(*b*) Note from the partial equation $Br_2 + 2e^- \rightarrow 2Br^-$ that two electrons react per Br_2. Thus, the equivalent weight of Br_2 is *half* its molecular weight, and 1 L of 1 N Br_2 requires (159.82/2) g = 79.91 g Br_2.

(*c*) One liter of 1 N H_3PO_4 requires (97.99/3) g = 32.66 g H_3PO_4.

12.20. Calculate the normality of each of the following solutions: (*a*) 7.88 g of HNO_3 per liter of solution, (*b*) 26.5 g of Na_2CO_3 per liter of solution (if acidified to form CO_2).

(*a*)
$$\text{equivalent weight of } HNO_3 = \text{formula weight} = 63.02$$
$$N = \text{normality} = \frac{(7.88\text{ g})/(63.02\text{ g/g-eq})}{1\text{ L}} = 0.1251\text{ g-eq/L} = 0.1251\text{ N}$$

(*b*)
$$\text{equivalent weight of } Na_2CO_3 = (1/2) \times (\text{formula weight}) = (1/2)(106.0) = 53.0$$
$$N = \frac{(26.5\text{ g})/(53.0\text{ g/g-eq})}{1\text{ L}} = 0.500\text{ N}$$

12.21. How many cm^3 of 2.00 M $Pb(NO_3)_2$ contain 600 mg Pb^{2+}?

One liter of 1 M $Pb(NO_3)_2$ contains 1 mol Pb^{2+}. Then 1 L of 2 M contains 2 mol Pb^{2+}, or 414 g Pb^{2+} per liter, or 414 mg Pb^{2+} per cm^3. Hence 600 mg Pb^{2+} is contained in

$$\frac{600\text{ mg}}{414\text{ mg/cm}^3} = 1.45\text{ cm}^3$$

of 2.00 M $Pb(NO_3)_2$.

12.22. How many kilograms of wet NaOH containing 12% water are required to prepare 60 L of 0.50 N solution?

One liter of 0.50 N NaOH contains $(0.50)(40\text{ g}) = 20\text{ g} = 0.020\text{ kg}$ NaOH. Then 60 L of 0.50 N NaOH contains

$$(60)(0.020\text{ kg}) = 1.20\text{ kg NaOH}$$

The wet NaOH contains $100\% - 12\% = 88\%$ pure NaOH. Then:

$$88\text{ kg pure NaOH} \quad \text{is contained in} \quad 100\text{ kg wet NaOH}$$
$$1\text{ kg pure NaOH} \quad \text{is contained in} \quad \frac{100}{88}\text{ kg wet NaOH}$$

and
$$1.20\text{ kg pure NaOH} \quad \text{is contained in} \quad (1.20)\left(\frac{100}{88}\text{ kg}\right) = 1.36\text{ kg wet NaOH}$$

Or, by proportion, letting x = mass of wet NaOH that gives 1.20 kg pure NaOH,

$$\frac{100\text{ kg wet}}{88\text{ kg pure}} = \frac{x}{1.20\text{ kg pure}} \qquad \text{or} \qquad x = 1.36\text{ kg wet NaOH}$$

12.23. Given the unbalanced equation

$$K^+MnO_4^- + K^+I^- + (H^+)_2SO_4^{2-} \rightarrow (K^+)_2SO_4^{2-} + Mn^{2+}SO_4^{2-} + I_2 + H_2O$$

(*a*) How many grams of $KMnO_4$ are needed to make 500 cm^3 of 0.250 N solution? (*b*) How many grams of KI are needed to make 25 cm^3 of 0.36 N solution?

(*a*) In this oxidation-reduction reaction, the oxidation state of Mn changes from +VII in MnO_4^- to +II in Mn^{2+}. Hence

$$\text{equivalent weight of } KMnO_4 = \frac{\text{formula weight}}{\text{oxidation state change}} = \frac{158}{5} = 31.6$$

Then 0.500 L of 0.250 N requires

$$(0.500\text{ L})(0.250\text{ g-eq/L})(31.6\text{ g/g-eq}) = 3.95\text{ g } KMnO_4$$

(*b*) The oxidation state of I changes from −I in I^- to 0 in I_2. Hence

$$\text{equivalent weight of KI} = \text{formula weight} = 166$$

Then 0.025 L of 0.36 N requires

$$(0.025\text{ L})(0.36\text{ g-eq/L})(166\text{ g/g-eq}) = 1.49\text{ g KI}$$

12.24. Given the unbalanced equation

$$K^+MnO_4^- + Mn^{2+}SO_4^{2-} + H_2O \rightarrow MnO_2 + (K^+)_2SO_4^{2-} + (H^+)_2SO_4^{2-}$$

How many grams of $KMnO_4$ are needed to make 500 cm^3 of 0.250 N solution?

In this oxidation-reduction reaction, the oxidation state of Mn changes from +VII in MnO_4^- to +IV in MnO_2. Hence

$$\text{equivalent weight of } KMnO_4 = \frac{\text{formula weight}}{\text{oxidation state change}} = \frac{158}{3} = 52.7$$

Then 0.500 L of 0.250 N requires

$$(0.500\ \text{L})\,(0.250\ \text{g-eq/L})\,(52.7\ \text{g/g-eq}) = 6.59\ \text{g}\ KMnO_4$$

Compare this with Problem 12.23(*a*).

DILUTION PROBLEMS

12.25. To what extent must a solution of concentration 40 mg $AgNO_3$ per cm^3 be diluted to yield one of concentration 16 mg $AgNO_3$ per cm^3?

Let V be the volume to which 1 cm^3 of the original solution must be diluted to yield a solution of concentration 16 mg $AgNO_3$ per cm^3. Because the amount of solute does not change with dilution,

$$\begin{aligned} \text{volume}_1 \times \text{concentration}_1 &= \text{volume}_2 \times \text{concentration}_2 \\ 1\ \text{cm}^3 \times 40\ \text{mg/cm}^3 &= V \times 16\ \text{mg/cm}^3 \end{aligned}$$

Solving, $V = 2.5$ cm^3. Each cm^3 of the original solution must be diluted to a volume of 2.5 cm^3.

Another Method

The amount of solute per cm^3 of diluted solution will be 16/40 as much as in the original solution. Hence $(40/16)$ cm^3 = 2.5 cm^3 of the diluted solution will contain as much solute as 1 cm^3 of the original solution.

Note that 2.5 is not the number of cm^3 of water to be added, but the final volume of the solution after water has been added to 1 cm^3 of the original solution. The dilution formula always gives answers in terms of the *total* volume of solution. If we can assume that there is no volume shrinkage or expansion on dilution, the amount of water to be added in this problem is 1.5 cm^3 per cm^3 of original solution. Because this assumption cannot always be made, the answers in the subsequent problems will be left in terms of the *total* volumes of the solutions.

12.26. To what extent must a 0.50 M $BaCl_2$ solution be diluted to yield one of concentration 20 mg Ba^{2+} per cm^3?

The original solution contains 0.50 mol of $BaCl_2$ or of Ba^{2+} per liter. The mass of Ba^{2+} in 0.50 mol is

$$(0.50\ \text{mol})\,(137.3\ \text{g/mol}) = 68.6\ \text{g}\ Ba^{2+}$$

Thus 0.50 M $BaCl_2$ contains 68.6 g Ba^{2+} per liter, or 68.6 mg Ba^{2+} per cm^3.

The problem now is to find the extent to which a solution of strength 68.6 mg Ba^{2+} per cm^3 must be diluted to yield one of concentration 20 mg Ba^{2+} per cm^3.

$$\begin{aligned} \text{volume}_1 \times \text{concentration}_1 &= \text{volume}_2 \times \text{concentration}_2 \\ 1\ \text{cm}^3 \times 68.6\ \text{mg/cm}^3 &= V \times 20\ \text{mg/cm}^3 \end{aligned}$$

Solving, $V = 3.43$ cm^3. Each cm^3 of 0.50 M $BaCl_2$ must be diluted with water to a volume of 3.43 cm^3.

12.27. A procedure calls for 100 cm^3 of 20% H_2SO_4, density 1.14 g/cm^3. How much of the concentrated acid, of density 1.84 g/cm^3 and containing 98% H_2SO_4 by weight, must be diluted with water to prepare 100 cm^3 of acid of the required strength?

The concentrations must first be changed from a mass basis to a volumetric basis, so that the dilution equation will apply.

$$\text{mass of } H_2SO_4 \text{ per cm}^3 \text{ of 20\% acid} = (0.20)(1.14\ g/cm^3) = 0.228\ g/cm^3$$
$$\text{mass of } H_2SO_4 \text{ per cm}^3 \text{ of 98\% acid} = (0.98)(1.84\ g/cm^3) = 1.80\ g/cm^3$$

Let V be the volume of 98% acid required for 100 cm^3 of 20% acid.

$$\text{volume}_1 \times \text{concentration}_1 = \text{volume}_2 \times \text{concentration}_2$$
$$100\ cm^3 \times 0.228\ g/cm^3 = V \times 1.80\ g/cm^3$$

Solving, $V = 12.7\ cm^3$ of the concentrated acid.

12.28. What volumes of N/2 and of N/10 HCl must be mixed to give 2 L of N/5 HCl?

Let x = volume of N/2 required; $2\text{ L} - x$ = volume of N/10 required.

$$\text{number of g-eq of N/5} = (\text{number of g-eq of N/2}) + (\text{number of g-eq of N/10})$$
$$(2\text{ L})\left(\frac{1}{5}\text{ N}\right) = x\left(\frac{1}{2}\text{ N}\right) + (2\text{ L} - x)\left(\frac{1}{10}\text{ N}\right)$$

Solving, $x = 0.5$ L. Thus 0.5 L of N/2 and 1.5 L of N/10 are required.

12.29. How many cubic centimeters of concentrated sulfuric acid, of density 1.84 g/cm^3 and containing 98% H_2SO_4 by weight, should be taken to make (*a*) one liter of normal solution, (*b*) one liter of 3.00 N solution, (*c*) 200 cm^3 of 0.500 N solution?

$$\text{equivalent weight of } H_2SO_4 = \tfrac{1}{2}(\text{formula weight}) = \tfrac{1}{2}(98.1) = 49.0$$

The H_2SO_4 content of 1 L of the concentrated acid is $(0.98)(1000\ cm^3)(1.84\ g/cm^3) = 1800\ g\ H_2SO_4$. The normality of the concentrated acid is

$$\frac{1800\ g\ H_2SO_4/L}{49.0\ g\ H_2SO_4/\text{g-eq}} = 36.7\ \text{g-eq/L}$$

The dilution formula, $V_{\text{conc.}} \times N_{\text{conc.}} = V_{\text{dil.}} \times N_{\text{dil.}}$, can now be applied to each case.

(*a*) $$V_{\text{conc.}} = \frac{(1\text{ L})(1.00\text{ N})}{36.7\text{ N}} = 0.0272\text{ L} = 27.2\ cm^3 \text{ conc. acid}$$

(*b*) $$V_{\text{conc.}} = \frac{(1\text{ L})(3.00\text{ N})}{36.7\text{ N}} = 0.0817\text{ L} = 81.7\ cm^3 \text{ conc. acid}$$

(*c*) $$V_{\text{conc.}} = \frac{(200\ cm^3)(0.500\text{ N})}{36.7\text{ N}} = 2.72\ cm^3 \text{ conc. acid}$$

Supplementary Problems

CONCENTRATIONS EXPRESSED IN PHYSICAL UNITS

12.30. How much NH_4Cl is required to prepare 100 cm^3 of a solution of strength 70 mg NH_4Cl per cm^3? *Ans.* 7.0 g

12.31. How many grams of concentrated hydrochloric acid, containing 37.9% HCl by weight, will give 5.0 g HCl? *Ans.* 13.2 g

12.32. It is required to prepare 100 g of a 19.7% by weight solution of NaOH. How many grams each of NaOH and H_2O are required? *Ans.* 19.7 g NaOH, 80.3 g H_2O

12.33. How much $CrCl_3 \cdot 6H_2O$ is needed to prepare 1 L of solution containing 20 mg Cr^{3+} per cm^3? *Ans.* 102 g

12.34. How many grams of Na_2CO_3 are needed to prepare 500 cm^3 of a solution containing 10.0 mg CO_3^{2-} per cm^3? *Ans.* 8.83 g

12.35. Calculate the volume occupied by 100 g of sodium hydroxide solution of density 1.20 g/cm^3. *Ans.* 83.3 cm^3

12.36. What volume of dilute nitric acid, of density 1.11 g/cm^3 and 19% HNO_3 by weight, contains 10 g HNO_3? *Ans.* 47 cm^3

12.37. How many cm^3 of a solution containing 40 g $CaCl_2$ per liter are needed to react with 0.642 g of pure Na_2CO_3? $CaCO_3$ is formed in the reaction. *Ans.* 16.8 cm^3

12.38. Ammonia gas is passed into water, yielding a solution of density of 0.93 g/cm^3 and containing 18.6% NH_3 by weight. What is the mass of NH_3 per cm^3 of solution? *Ans.* 173 mg/cm^3

12.39. Hydrogen chloride gas is passed into water, yielding a solution of density 1.12 g/cm^3 and containing 30.5% HCl by weight. What is the mass of HCl per cm^3 of solution? *Ans.* 342 mg/cm^3

12.40. Given 100 cm^3 of pure water at 4 °C, what volume of a solution of hydrochloric acid, density 1.175 g/cm^3 and containing 34.4% HCl by weight, could be prepared? *Ans.* 130 cm^3

12.41. A volume of 105 cm^3 of pure water at 4 °C is saturated with NH_3 gas, yielding a solution of density 0.90 g/cm^3 and containing 30% NH_3 by weight. Find the volume of the ammonia solution resulting, and the volume of the ammonia gas at 5 °C and 775 torr which was used to saturate the water. *Ans.* 167 cm^3, 59 L

12.42. An excellent solution for cleaning grease stains from cloth or leather consists of the following: carbon tetrachloride 80% (by volume), ligroin 16%, amyl alcohol 4%. How many cm^3 of each should be taken to make up 75 cm^3 of solution? (Assume no volume change on mixing.) *Ans.* 60 cm^3, 12 cm^3, 3 cm^3

12.43. A liter of milk weighs 1032 g. The butterfat which it contains to the extent of 4.0% by volume has a density of 0.865 g/cm^3. What is the density of the fat-free "skimmed" milk? *Ans.* 1.039 g/cm^3

12.44. To make a benzene-soluble cement, melt 49 g of rosin in an iron pan and add 28 g each of shellac and beeswax. How much of each component should be taken to make 75 kg of cement? *Ans.* 35 kg rosin, 20 kg shellac, 20 kg beeswax

12.45. How much $CaCl_2 \cdot 6H_2O$ and water must be weighed out to make 100 g of a solution that is 5.0% $CaCl_2$? *Ans.* 9.9 g $CaCl_2 \cdot 6H_2O$, 90.1 g water

12.46. How much $BaCl_2$ would be needed to make 250 cm^3 of a solution having the same concentration of Cl^- as one containing 3.78 g NaCl per 100 cm^3? *Ans.* 16.8 g $BaCl_2$

CONCENTRATIONS EXPRESSED IN CHEMICAL UNITS

12.47. What is the molar concentration of a solution containing 37.5 g $Ba(MnO_4)_2$ per liter, and what is the molar concentration with respect to each type of ion?
Ans. 0.100 M $Ba(MnO_4)_2$, 0.100 M Ba^{2+}, 0.200 M MnO_4^-

12.48. How many grams of solute are required to prepare 1 L of 1 M $CaCl_2 \cdot 6H_2O$? *Ans.* 219.1 g

12.49. Exactly 100 g of NaCl is dissolved in sufficient water to give 1500 cm^3 of solution. What is the molar concentration? *Ans.* 1.14 M

12.50. Calculate the molality of a solution containing (*a*) 0.65 mol glucose, $C_6H_{12}O_6$, in 250 g water, (*b*) 45 g glucose in 1 kg water, (*c*) 18 g glucose in 200 g water.
Ans. (*a*) 2.6 molal; (*b*) 0.25 molal; (*c*) 0.50 molal

12.51. How many grams of $CaCl_2$ should be added to 300 mL of water to make up a 2.46 molal solution? *Ans.* 82 g

12.52. A solution contains 57.5 cm^3 of ethyl alcohol (C_2H_5OH) and 600 cm^3 of benzene (C_6H_6). How many grams of alcohol are in 1000 g benzene? What is the molality of the solution? Density of C_2H_5OH is 0.80 g/cm^3; of C_6H_6, 0.90 g/cm^3. *Ans.* 85 g, 1.85 mol/kg

12.53. A solution contains 10.0 g of acetic acid, CH_3COOH, in 125 g of water. What is the concentration of the solution expressed as (*a*) mole fractions of CH_3COOH and H_2O, (*b*) molality?
Ans. (*a*) x(acid) = 0.024, x(water) = 0.976; (*b*) 1.33 mol/kg

12.54. A solution contains 116 g acetone (CH_3COCH_3), 138 g ethyl alcohol (C_2H_5OH), and 126 g water. Determine the mole fraction of each.
Ans. x(acetone) = 0.167, x(alcohol) = 0.250, x(water) = 0.583

12.55. What is the mole fraction of the solute in a 1.00 molal aqueous solution? *Ans.* 0.0177

12.56. An aqueous solution labeled 35.0% $HClO_4$ had a density 1.251 g/cm^3. What are the molar concentration and molality of the solution? *Ans.* 4.36 M, 5.36 mol/kg

12.57. A sucrose solution was prepared by dissolving 13.5 g $C_{12}H_{22}O_{11}$ in enough water to make exactly 100 cm^3 of solution, which was then found to have a density of 1.050 g/cm^3. Compute the molar concentration and molality of the solution. *Ans.* 0.395 M, 0.431 mol/kg

12.58. For a solute of molecular weight W, show that the molar concentration M and molality m of the solution are related by

$$M\left(\frac{W}{1000} + \frac{1}{m}\right) = d$$

where d is the solution density in g/cm^3. (*Hint*: Show that each cm^3 of solution contains $MW/1000$ grams of solute and M/m grams of solvent.) Use this relation to check the answers to Problems 12.56 and 12.57.

12.59. What volume of a 0.232 N solution contains (*a*) 3.17 meq of solute, (*b*) 6.5 g-eq of solute?
Ans. (*a*) 13.7 cm^3; (*b*) 28.0 L

12.60. Determine the molar concentration of each of the following solutions: (*a*) 166 g KI per liter of solution, (*b*) 33.0 g $(NH_4)_2SO_4$ in 200 cm^3 of solution, (*c*) 12.5 g $CuSO_4 \cdot 5H_2O$ in 100 cm^3 of solution, (*d*) 10.0 mg Al^{3+} per cm^3 of solution.
Ans. (*a*) 1.00 M; (*b*) 1.25 M; (*c*) 0.500 M; (*d*) 0.371 M

12.61. What volume of 0.200 M $Ni(NO_3)_2 \cdot 6H_2O$ contains 500 mg Ni^{2+}? *Ans.* 42.6 cm^3

12.62. Compute the volume of concentrated H_2SO_4 (density 1.835 g/cm^3, 93.2% H_2SO_4 by weight) required to make up 500 cm^3 of 3.00 N acid. *Ans.* 43.0 cm^3

12.63. Compute the volume of concentrated HCl (density 1.19 g/cm^3, 38% HCl by weight) required to make up 18 L of N/50 acid. *Ans.* 29 cm^3

12.64. Determine the mass of $KMnO_4$ required to make 80 cm^3 of N/8 $KMnO_4$ when the latter acts as an oxidizing agent in acid solution and Mn^{2+} is a product of the reaction. *Ans.* 0.316 g

12.65. Given the unbalanced equation

$$Cr_2O_7^{2-} + Fe^{2+} + H^+ \rightarrow Cr^{3+} + Fe^{3+} + H_2O$$

(*a*) What is the normality of a $K_2Cr_2O_7$ solution 35.0 cm^3 of which contains 3.87 g of $K_2Cr_2O_7$? (*b*) What is the normality of a $FeSO_4$ solution 750 cm^3 of which contains 96.3 g of $FeSO_4$? *Ans.* (*a*) 2.25 N; (*b*) 0.845 N

12.66. What mass of $Na_2S_2O_3 \cdot 5H_2O$ is needed to make up 500 cm^3 of 0.200 N solution for the following reaction?

$$2S_2O_3^{2-} + I_2 \rightarrow S_4O_6^{2-} + 2I^-$$

Ans. 24.8 g

DILUTION PROBLEMS

12.67. A solution contains 75 mg NaCl per cm^3. To what extent must it be diluted to give a solution of concentration 15 mg NaCl per cm^3 of solution?
Ans. Each cm^3 of original solution is diluted with water to a volume of 5 cm^3.

12.68. How many cm^3 of a solution of concentration 100 mg Co^{2+} per cm^3 are needed to prepare 1.5 L of solution of concentration 20 mg Co^{2+} per cm^3? *Ans.* 300 cm^3

12.69. Calculate the approximate volume of water that must be added to 250 cm^3 of 1.25 N solution to make it 0.500 N (neglecting volume changes). *Ans.* 375 cm^3

12.70. Determine the volume of dilute nitric acid (density 1.11 g/cm^3, 19.0% HNO_3 by weight) that can be prepared by diluting with water 50 cm^3 of the concentrated acid (density 1.42 g/cm^3, 69.8% HNO_3 by weight). Calculate the molar concentrations and molalities of the concentrated and dilute acids.
Ans. 235 cm^3; molar concentrations, 15.7 and 3.35; molalities, 36.7 and 3.72

12.71. What volume of 95.0% alcohol by weight (density 0.809 g/cm^3) must be used to prepare 150 cm^3 of 30.0% alcohol by weight (density 0.957 g/cm^3)? *Ans.* 56.0 cm^3

12.72. What volumes of 12 N and 3 N HCl must be mixed to give 1 L of 6 N HCl?
Ans. $\frac{1}{3}$ liter 12 N, $\frac{2}{3}$ liter 3 N